# ACADEMIC DISCIPLINE: MATHEMATICS & COMPUTATIONAL THINKING

Table 1: AD Major Structure for Bachelor of Science in Mathematics & Computational Thinking and Education (Primary / Secondary)

Year	Course Code	Title	Course Category	No. of AUs	Pre- requisites		
1	AAM10A	Linear Algebra I	Core	3	-		
	AAM10B	Calculus I	Core	3	-		
	AAM10E	Introduction to Probability and Statistics	Core	3	-		
	AAM10G	Discrete Mathematics	Core	3	-		
2	AAM20A	Linear Algebra II	Core	3	Must have done AAM10A		
	AAM20B	Calculus II	Core	3	-		
	AAM20C	Statistics I	Core	3	-		
	AAM20D	Computational Mathematics	Core	3	-		
	AAM20E	Differential Equations	Core	3	-		
	AAM20G	Complex Analysis	Core	3	-		
	Select any 3 electives						
3	AAM33A	Special Topics in Mathematics I	Pres	3	-		
	AAM33C	Statistics II	Pres	3	-		
	AAM33D	Real Analysis	Pres	3	-		
	AAM33E	Modern Algebra	Pres	3	-		
	AAM33G	Modelling with Differential Equations	Pres	3	AAM20E		
	AAM33J	Combinatorial Analysis	Pres	3	-		
	Select any 3 electives*						
4	AAM43A	Special Topics in Mathematics II	Pres	3	-		
	AAM43B	Statistical Theory	Pres	3	AAM33C		
	AAM43C	Applied Statistics	Pres	3	AAM33C		
	AAM43D	Techniques in Operations Research	Pres	3	-		
	AAM43E	Mathematical Programming and Stochastic Processes	Pres	3	-		
	AAM43G	Metric Spaces	Pres	3	-		
	AAM43H	Galois Theory	Pres	3	AAM33E		
	AAM43J	Graph Theory	Pres	3	-		
	AAM43K	Geometry	Pres	3	-		
	AAM43L	Advanced Mathematical Modelling	Pres	3	AAM20E		

Please refer to the NIE Portal for the list of courses offered by semesters.

<sup>\*</sup> Student teachers reading AD Mathematics & Computational Thinking may replace one Year 4 course with one Year 3 course that they have not previously taken.

# Minor in Mathematics & Computational Thinking (NIE)

Year	Course Code	Title	Course Category	No. of AUs	Pre- requisites
1	AAM10A	Linear Algebra I	Core	3	-
	AAM10B	Calculus I	Core	3	-
	AAM10E	Introduction to Probability and Statistics	Core	3	-
	AAM10G	Discrete Mathematics	Core	3	-

For details of the courses listed above, please refer to AD Major: Mathematics & Computational Thinking.

#### **AAM10A Linear Algebra I**

This first course in linear algebra aims to provide student teachers with the knowledge of basic set theory, functions, linear systems, matrix algebra, determinant function, eigenvalues and eigenvectors, and vectors in 2-space and 3-space. It will equip student teachers with the content knowledge for taking other courses, especially Linear Algebra II, which is the second course in linear algebra. It also provides student teachers with the opportunities to see the interlink between different structures in mathematics. Topics covered include: Basic set theory, functions, linear systems and solutions, Gaussian and Gauss-Jordan eliminations, matrices and matrix operations, invertible matrices, methods of finding inverse of an determinant function invertible matrix. applications, vectors in 2-space and 3-space, dot product, cross product, equations of straight lines and eigenvalues and eigenvectors. planes. diagonalisation.

#### **AAM10B Calculus I**

This course aims to introduce student teachers to concepts in beginning undergraduate calculus (of one variable) so that student teachers can have a deeper understanding of school calculus content knowledge and the advanced knowledge of calculus so that they are prepared for Multivariable calculus and Real Analysis. This course also aims to give student teachers an introductory calculus related proofs in εdefinitions. Topics covered include: δ and ε-N Functions, sequences and graphs, limits of functions, and  $\epsilon$ -N definitions, Continuous ε-δ definitions functions and results (Intermediate Value Theorem), Derivative, techniques Differentiation and differentiation and their results (Mean value Theorem and Rolle's Theorem), Applications of Differentiation in gradients of tangents and normal, analysis of functions, Riemann sums and Riemann integral, Antiderivative, Indefinite Integral Theorem and Fundamental Theorem of Calculus, Integration Techniques.

## **AAM10E Introduction to Probability and Statistics**

This course aims to introduce you to the fundamental concepts and methods of probability and statistics, equipping you with the skills to perform effective data analysis and extract meaningful insights. Additionally, it provides a solid foundation for further study in advanced statistical topics and their applications. Topics covered include: Measures of location and variability. Discrete and continuous data. Sample space and events. Conditional probability and independence. Discrete and continuous probability distributions. Mathematical expectations. Sampling distributions and Central Limit Theorem. Hypothesis testing: one sample for mean.

#### **AAM10G Discrete Mathematics**

This course aims to introduce you to basic counting principles and techniques and to develop your mathematical problem-solving skills by applying these principles and techniques to solve a variety of counting problems. It also aims to introduce you to fundamental properties of integers, and to mathematical reasoning and methods of proofs by using integers as a context. Topics covered include: Addition and Multiplication Principles, permutations and combinations, Bijection Principle, distributions, generalized permutations and combinations. Theorem. **Principle** Binomial Inclusion and Exclusion (for up to three sets), Pigeonhole Principle (basic version), methods of proof, axioms of Integers, divisibility, greatest common divisors, Euclidean algorithm, Diophantine equations, primes and the Fundamental Theorem of Arithmetic.

### **AAM20A Linear Algebra II**

This is a second course in linear algebra, which builds on the concepts learned in Linear Algebra I. It aims to student teachers to the fundamental introduce real spaces concepts of vector and transformations. The materials covered in this core course, as well as in Linear Algebra I, form essential background for future study in all other areas of pure and applied mathematics. Topics covered include: Basic logic, real vector spaces, subspaces, linear independence, spanning sets, basis and dimension, row space, column space, nullspace, rank and nullity, orthogonality in Euclidean n-space, eigenvalues and eigenvectors, diagonalization, linear transformations.

#### **AAM20B Calculus II**

This is the second core course in calculus, which builds on the concepts learned in Calculus I. It aims to introduce student teachers to sequences and series, and the calculus of real-valued functions of two variables. Topics covered include Sequences, ε-N definition of limit of sequence, limit theorems for sequences, monotonic and bounded sequences, infinite series, definition of convergence and sum of infinite series, various tests for convergence and divergence of series, absolute and conditional convergence, power series, interval and radius of convergence, differentiation and integration of power series, Taylor series and Maclaurin series. Partial for functions derivatives of two differentiability, and chain rules for functions of two variables, directional derivatives and gradients for functions of two variables, tangent planes, linearization, maxima and minima of functions of two variables, double integrals.

#### **AAM20C Statistics I**

This first course in statistics aims to introduce student teachers to the fundamental concepts in statistics. The course will provide a good foundation for study in further statistics topics and applications. Topics covered include Discrete and continuous distributions. Mathematical expectations. Sampling distributions and Central Limit Theorem. Estimation, confidence intervals and hypothesis testing: one sample for mean, proportion and variance.

#### **AAM20D Computational Mathematics**

This course aims to teach student teachers how to exploit the power of modern computers as an experimental adjunct to support their theoretical understanding of mathematics. The course will cover the following suggested topics but with the option to substitute with other topics in mathematics where computing could be applied: Arithmetic and Number Theory: distribution of primes in the large number limit. Calculus and Analysis: graph plotting, numerical differentiation, and integration. Geometry and Algebra: operations, vector and matrix transformations, fractals. Probability and Statistics: random numbers, simulation, the normal distribution. Other Programming Languages: appreciation of commonality of core concepts

# **AAM20E Differential Equations**

This course aims to introduce student teachers to elementary theory and application of (ordinary) differential equations so that student teachers can

have a deeper understanding of A-level differential equations and the advanced knowledge of ordinary differential equations so that they are prepared for advanced theory and modelling using differential equations. Topics covered include Qualitative Theory of First Order Differential equations: Existence and Uniqueness Theorem, Slope fields. Analytic solution of some common first order differential equations (separable, linear, exact) and others that are reducible to the above forms by substitution. Modelling of some real-world phenomena by first order differential (including population growth equations models. compartmental analysis, Newtonian mechanics of motion under variable forces). Theory of second order differential equations (including liner dependence and independence of solutions to homogeneous linear differential equations and Wronskian). Solution of order linear homogeneous differential equations with constant coefficients and other types of equations reducible to this (e.g. Euler's equation). of order non-homogeneous Solution second differential equations by method of undetermined coefficient and variation of parameters. Modelling of some real world phenomena by second order differential equations (including simple harmonic motion).

# **AAM20G Complex Analysis**

This course deals with the theory of functions of one complex variable. The main objective of this course is to acquaint student teachers to complex valued functions of a special kind, namely the holomorphic (or analytic) functions. In this course, student teachers will learn more about the complex number system, its Euclidean topology as well as the theory of analytic functions. One ultimate goal is that student teachers are able to make use of complex integration and prove

the Fundamental Theorem of Algebra. Topics covered include: Complex numbers in various representations, basic topology on the complex plane, limits and continuity, differentiability and analyticity, Cauchy-Riemann equations, examples of complex valued (polynomials, trigonometrical functions, functions of logarithmic exponential functions. branches integrals. Cauchy-Goursat functions). Contour Theorem, Deformation Principle, Cauchy Integral Formula, Residue Theorem, Fundamental Theorem of Algebra.

### **AAM33A Special Topics in Mathematics I**

This course aims to introduce student teachers to some selected topics in mathematics so that they are exposed to different fields of mathematics (which may include real-life applications of mathematics) that are not seen in the regular Year 3 course offerings.

#### **AAM33C Statistics II**

This course in statistics is a continuation of the statistics course AAM20C. The course aims to provide a good foundation for study in further statistics topics and applications. Topics covered include: Two-sample confidence intervals and two-sample hypothesis testing for comparing means, proportions or variances; Chi-square goodness-of-fit tests, and contingency tables: test of homogeneity and test of independence; Simple linear regression: least squares estimation and inference (including diagnostic checking).

### **AAM33D Real Analysis**

This course deals with the properties of real numbers in three aspects: order, algebra and topology. It also deals with fundamental notions of limits, continuity, differentiability and Riemann-integrability. The main

objective of this course is to acquaint student teachers to real number system and the real-valued functions. In this course, student teachers will learn more about (1) the real number system: the completeness axiom, the Nested Interval Property, and the Bolzano-Weierstrass Theorem, and (2) properties of realvalued functions. One ultimate goal is that student teachers are able to prove the Fundamental Theorem **Topics** covered Calculus. include: of Completeness Axiom. The Archimedean Property. Density of rational and irrationals. The limit of a sequence, limit theorems. The limit of a function, the continuity of a function. The Intermediate Value **Bolzano-Weierstrass** Theorem. The Theorem. Extreme Value Theorem. Differentiation. Mean Value Theorem. Riemann Integration. The Fundamental Theorem of Calculus.

### **AAM33E Modern Algebra**

This first course in abstract algebra aims to introduce student teachers to the algebraic structures of rings and groups, and to present a range of examples to facilitate the understanding of the abstract theory so that student teachers have a good grasp of the fundamental concepts in abstract algebra. This course will provide a good foundation for further study in advanced algebra topics and in areas where abstract algebra has applications. Topics covered include Rings and subrings, integral domains and fields, ring homomorphism, isomorphism and rings polynomials, divisibility in polynomial rings over a field, factorization of polynomials over a field, ideals and quotient rings of commutative rings with identity. First Isomorphism Theorem. Groups and subgroups, cyclic groups, permutations, symmetric group on n letters, cosets, Lagrange's Theorem, quotient groups, group isomorphism and homomorphism, Fundamental Homomorphism Theorems.

## **AAM33G Modelling with Differential Equations**

The course aims to equip student teachers with the skills and knowledge of solving differential equations using specific methods with a view to apply them in investigating real world problems through mathematical models. The course consists of five major topics: The Laplace Transform and its use in solving ordinary differential equations. Introduction to MATLAB and its application to mathematical problem solving. Numerical Solutions of Ordinary Differential Equations. Systems of Linear First-order Differential Equations. Plane Autonomous Systems.

#### **AAM33J Combinatorial Analysis**

This course aims to develop student teachers' mathematical problem-solving skills by a variety of counting problems which can be solved by the general inclusion-exclusion generating principle of or functions. Topics covered include: Principle of inclusion-exclusion, general principle of inclusionexclusion, Surjective mappings, the Stirling number of the second kind, derangement and Euler φ-functions; ordinary generating functions, operations of ordinary generating functions; some modelling problems, partitions of integers, and exponential generating functions.

## **AAM43A Special Topics in Mathematics II**

This course aims to introduce student teachers to some selected advanced topics in mathematics so that they are exposed to different fields of mathematics (which may include real-life applications of mathematics) that are not seen in the regular Year 4 course offerings.

# **AAM43B Statistical Theory**

This course aims to develop an understanding of the mathematical foundations of statistical inference. Topics covered include: Further univariate distributions; Bivariate distributions; Moment generating functions and proof of Central Limit Theorem; Sampling distributions: t-, F-, and chi-square distributions; Selected topics from estimation theory and hypothesis testing theory.

## **AAM43C Applied Statistics**

This course in statistics provides a continuation of the two introductory statistics courses AAM20C and AAM33C. This course aims to further introduce a range of statistical methods and applications on design of experiments and non-parametric tests.

Topics covered include: Elements of a designed experiment; One-factor analysis of variance and experimental designs with emphasis on completely randomised design and randomised block design, multiple comparison, residual analysis and model checking; Two-factor analysis of variance and factorial designs, multiple comparison; Non-parametric tests with emphasis on signed-rank test, Wilcoxon rank-sum test, Kruskal-Wallis test, and Friedman test.

#### **AAM43D Techniques in Operations Research**

This course aims to provide an introduction to certain topics in the field of Operations Research, called "OR" for short, where mathematical methods are applied to real world problems which possess quantitative solutions. This course will focus on two of the four principal pillars of OR, which are methods which apply

to networks, and those which apply to more general routing or scheduling problems. The other two pillars are covered in the standalone course AAM43E Programming Mathematical and Stochastic Processes. This course will cover the following suggested topics: 1) Network Optimization: shortest spanning trees, trees of shortest paths, sensitivity analysis. 2) Transportation Problems: maximal flows, the Ford-Fulkerson algorithm, least cost flows. 3) Scheduling and Sequencing: critical path analysis and the job shop scheduling problem. 4) Routing and Allocation: least cost assignments and the travelling salesman problem.

#### **AAM43E Mathematical Programming and Stochastic Processes**

This course aims to provide an introduction to certain topics in the field of Operations Research, called "OR" for short, where mathematical methods are applied to real world problems which possess quantitative solutions. This course will focus on two of the four principal pillars of OR, which are methods applied to systems of linear equations, and those which apply to probabilistic events and queueing systems. The other two pillars are covered in the standalone course AAM43D Techniques in Operations Research. This course will cover the following suggested topics: 1) Processes: random arrivals Stochastic and departures, application to inventory systems. Queueing Theory: single and multi-server queueing systems, queue length, waiting time. 3) Linear Programming: simplex simplex the and dual algorithms. sensitivity analysis. Integer 4) Programming: inclusion of integer constraints, branch and bound algorithms.

#### **AAM43G Metric Spaces**

This course is an introductory one for metric spaces. Metric spaces are a generalization of Euclidean distance that is commonly used in calculus of real and complex variables. Theorems in metric space theory are useful in establishing principles of convergence commonplace in real and complex analyses. Topics covered include: Introduction to metric spaces; Open sets and closed sets; Sequences in metric spaces; Continuity; Compactness and completeness; Contraction mapping.

#### **AAM43H Galois Theory**

This course aims to introduce student teachers to Galois theory, which brings together different areas of mathematics to solve some classical mathematical problems. It explains why there are formulas for the roots of quadratic, cubic and quartic equations, but no formulas exist for the roots of polynomial equations of degree 5 and above. Topics covered include: Field extensions, simple, finite and algebraic extensions, splitting fields, normal and separable extensions, primitive elements, finite fields, Galois groups, Galois extensions, The Fundamental Theorem of Galois Theory, solvability by radicals.

### **AAM43J Graph Theory**

This course aims to provide student teachers with some fundamental knowledge of graph theory, to train student teachers to apply graph theory to solve other problems, and to develop student teachers' ability to write solutions with logical explanations. Topics covered include: Fundamental concepts and basic results (including Handshaking Lemma and the triangular inequality for distances); Graph isomorphism, testing isomorphic graphs, subgraphs,

complements of graphs, self-complementary graphs; bipartite graphs; tree and its characterizations; spanning trees of graphs; the four-color theorem; vertex colouring; chromatic number; enumeration of chromatic numbers; greedy colouring algorithm; Brooks' theorem; application of colourings.

## **AAM43K Geometry**

Geometry is one of the most fundamental and important mathematics topics. The modern Euclidean geometry was built as an axiomatic system. This course will introduce a complete and rigorous axiomatic system for Euclidean plane geometry. It will also briefly cover the non-Euclidean geometries such as Elliptic geometry and Hyperbolic geometry. By taking this course, student teachers will get a deeper understanding of the axiomatic structures of plane geometry, know the undefined and defined terms, axioms and main theorems. They will also learn the rigour and correct concepts in plane geometry, such as the definition of line segment, ray, angle, triangle, congruency and similarity of triangles, angle measure and area. Topics covered include: Axiomatic systems; Incidence Geometry; undefined terms and axioms of Neutral geometry; the three parallel postulations; Euclidean geometry; tests for congruent triangles; alternate interior angles theorem and conditions for parallel lines; similar triangles; the Fundamental Theorem for similar triangles; the Pythagorean Theorem; the Median concurrence Theorem; Non-Euclidean Geometry: Hyperbolic Geometry; the Neutral Area postulate; the Euclidean Area postulate; area and defect in hyperbolic geometry.

#### **AAM43L Advanced Mathematical Modelling**

The course aims to equip student teachers with basic knowledge in partial differential equations and their solutions using numerical methods, with a view to applying them in investigating and examining mathematical models such as the diffusion equation, Laplace equation and wave equation. The course consists of five major topics: 1) Fourier Series and its use in representation of periodic functions. 2) Introduction to partial differential equations. 3) Analytic method of solving partial differential equations. 4) Numerical methods for partial differential equations. 5) Convergence and Stability of numerical methods.