Annexe A: New/Revised Course Content in OBTL+ Format

Course Overview

Expected Implementation in Academic Year	AY2025-2026
Semester/Trimester/Others (specify approx. Start/End date)	Semester 2
Course Author * Faculty proposing/revising the course	Cheong Siew Ann
Course Author Email	cheongsa@ntu.edu.sg
Course Title	Complex Methods for the Sciences
Course Code	MH2801
Academic Units	3
Contact Hours	38
Research Experience Components	Not Applicable

Course Requisites (if applicable)

Pre-requisites	(MH1101 & MH1200) or (MH1802 & MH1803 & MH1200) or (MH1802 & MH1803 & MH2802) or (CY1601 & CY1602)
Co-requisites	None
Pre-requisite to	PH3101
Mutually exclusive to	MH3101
Replacement course to	None
Remarks (if any)	None

Course Aims

This course introduces the use of mathematical techniques based on complex numbers and their applications in physics and the other sciences. The techniques include contour integration, Fourier transforms, and Green's functions, and the applications include the solution of definite integrals and differential equations, and the modelling and analysis of oscillators and waves.

Course's Intended Learning Outcomes (ILOs)

Upon the successful completion of this course, you (student) would be able to:

ILO 1	Manipulate algebraic expressions involving complex numbers to calculate the real part, imaginary part, magnitude, argument, and related quantities.
ILO 2	Use Euler's formula to translate between complex exponentials, trigonometric functions, and hyperbolic functions, and to solve integrals containing combinations of such functions.
ILO 3	Use the complex plane to visualise complex numbers and simple operations acting on them.
ILO 4	Formulate the branch functions for a multi-valued operation based on roots and/or the complex logarithm, including identifying branch points and selecting branch cuts.
ILO 5	Plot the contour, or complex plane trajectory, of a complex function of a real variable.
ILO 6	Use complex numbers to solve harmonic oscillator and wave problems.
ILO 7	Identify the domains of analyticity for common complex functions such as reciprocals.
ILO 8	Use the Cauchy-Riemann equations to determine if a given complex function is analytic, or to reconstruct an analytic function from its real or imaginary part, and related tasks.
ILO 9	Determine the simple poles and residues of a complex function.
ILO 10	Evaluate contour integrals via parameterisation and via Cauchy's integral theorem.
ILO 11	Solve definite integrals via contour integration, the calculus of residues, and Jordan's lemma.
ILO 12	Calculate the Fourier series coefficients of a periodic function of a real variable.
ILO 13	Calculate the Fourier transform of a square-integrable function of a real variable.
ILO 14	Deduce the features of a function from its Fourier spectrum, and vice versa.
ILO 15	Use Fourier transforms to solve linear differential equations.
ILO 16	Derive the Green's function for a driven oscillator or wave problem, and use it to obtain the solution for an arbitrary driving source.

Course Content

Review of Real Functions
- Exponential, logarithm, trigonometric, and hyperbolic functions
- Power operations
- Definition of real derivatives and integrals
- Basic integration techniques (integration by parts and change of variables)
- The Gaussian integral

Complex Algebra
- Definition of complex numbers; basic complex algebraic manipulations; magnitude and argument of a complex number
- Complex exponential and trigonometric functions
- Euler's formula and its uses (e.g. for solving definite integrals)
- The complex plane and trajectories in the complex plane (contours)

Complex Oscillators and Waves
- Formulation and solution of the complex damped harmonic oscillator equation
- Interpretation of complex frequencies
- Under-damped, critically-damped, and over-damped motion

Complex Derivatives
- Definition of complex differentiability; the domain of analyticity of a complex function
- Derivation of the Cauchy-Riemann Equations and their applications

Branch Cuts and Branch Points
- Root and logarithms as multiple-valued operations
- Branch points of root and logarithm operations
- Branch cuts and the formulation of branch functions

Contour Integrals
- Evaluation of contour integral via parameterisation
- Cauchy's Integral Theorem
- The calculus of residues and Jordan's lemma
- Cauchy's principal value integrals

Fourier Transforms
- The Fourier series and its properties
- The Fourier Transform and Inverse Fourier Transform
- Interpretation of Fourier spectra

Green's Functions
- Formulation and application of the Green's function for a damped driven harmonic oscillator
- Formulation and application of the Green's function for waves in empty space
- Causality and the Green's function in space and time

Reading and References (if applicable)

1. Y. D. Chong, Complex Methods for the Sciences (online notes)

Planned Schedule

Week or Session	Topics or Themes	ILO	Readings	Delivery Mode	Activities
1	Review of complex numbers; Cartesian and polar representations, Euler formula; trigonometric, hyper-trigonometric functions, and their inverses	1-3		In-person	Problem Set 1
2	Visualization of complex numbers on Argand diagram; visualizing functions of a single complex variable; multi-valuedness, Riemann surfaces	4		In-person	Problem Set 2
3	Branch points and branch cuts; review differentiating function of a single real variable; differentiating function of a single complex variable; Cauchy-Riemann relations	4-8		In-person	Problem Set 2, Problem Set 3

Week or Session	Topics or Themes	ILO	Readings	Delivery Mode	Activities
4	Contour integrals, open and closed; Cauchy integral theorem; Cauchy integral formula	5-11		In-person	Problem Set 4
5	Laurent series; classification of singularities; residue theorem; Cauchy principal value	9-10		In-person	Problem Set 5
6	Applications of contour integration to evaluate real integrals; Jordan lemma	11		In-person	Problem Set 6
7	Periodic functions; real Fourier series; complex Fourier series	12		In-person	Problem Set 7
8	Aperiodic functions; Fourier transforms; inverse Fourier transforms; evaluating inverse Fourier transforms using contour integration	13		In-person	Midterm Test, Problem Set 8
9	Dirac delta function; convolution	14		In-person	Problem Set 9
10	Applications of Fourier series to solve differential equations	15		In-person	Problem Set 10

Week or Session	Topics or Themes	ILO	Readings	Delivery Mode	Activities
11	Applications of Fourier transforms to solve differential equations	15		In-person	Problem Set 10
12	Green functions	16		In-person	Problem Set 11
13	Application of Green functions to solve differential equations	16		In-person	Problem Set 11

Learning and Teaching Approach

Approach	How does this approach support you in achieving the learning outcomes?
Lectures	The lectures build up the central concepts of the course, and provide concrete examples for calculations of the type and difficulty you are expected to be able to do.
Collaborative Problem Solving	During tutorial hours, you will come to the whiteboard to contribute partial solutions to selected problems. The approach is collaborative, because once previous students decided on how to solve the problem, subsequent students will need to make contributions following these leads. After experimenting with this approach during Sem 2 of AY 2024/2025, I found very high attendance rates for the tutorials. Because you will need to constantly pay attention to the developing solutions before you can contribute, during the test and exam I found that you are able to follow through multi-step answers on your own.

Assessment Structure

Assessment Components (includes both continuous and summative assessment)

No.	Component	ILO	Related PLO or Accreditation	Weightage	Description of Assessment Component	Team/Individual	Rubrics	Level of Understanding
1	Summative Assessment (EXAM): Final exam(Final Examination)	All		60	The final exam serves to check all intended learning outcomes. In particular, you should be able to understand the nature of a problem relevant to MH2801, and develop a multistep strategy to solve it, even if not all steps can be carried out to completion.	Individual	Analytic	Multistructural

No.	Component	ILO	Related PLO or Accreditation	Weightage	Description of Assessment Component	Team/Individual	Rubrics	Level of Understanding
2	Continuous Assessment (CA): Class Participation(CA1: Collaborative Problem Solving)	All		15	Mathematical methods is not a spectator sports. It is important for you to practice problem solving extensively. Therefore, instead of the traditional tutorial where a teaching assistant explains the steps needed to solve different problems, you will take turns with other students to solve problems collaboratively, one step at a time.	Individual	Analytic	Multistructural

No.	Component	ILO	Related PLO or Accreditation	Weightage	Description of Assessment Component	Team/Individual	Rubrics	Level of Understanding
3	Continuous Assessment (CA): Test/Quiz(CA2: Midterm Test)	1-11		20	Collaborative problem solving involves you working out one step at a time. To master the methods taught in this course, it is important to check that you can work through the multiple steps needed to solve problems. This is the purpose of the midterm test.	Individual	Analytic	Multistructural
4	Continuous Assessment (CA): Class Participation(You are invited to ask questions during two dedicated time slots in the lectures, so that you can clarify misconceptions at the earliest opportunity.)	All		5	I will invite 5 questions during each of the dedicated Q&A sessions during the lecture. You will get 1 mark for asking for a question during a lecture.	Individual	Holistic	Multistructural

Description of Assessment Components (if applicable)

Formative Feedback

You will receive formative feedback through discussion within tutorial lessons and via written feedback on graded assignments and the midterm. Solutions will be provided for assignments and the midterm.

NTU Graduate Attributes/Competency Mapping

This course intends to develop the following graduate attributes and competencies (maximum 5 most relevant)

Attributes/Competency	Level
Creative Thinking	Advanced
Problem Solving	Advanced

Course Policy

Policy (Academic Integrity)

Good academic work depends on honesty and ethical behaviour. The quality of your work as a student relies on adhering to the principles of academic integrity and to the NTU Honour Code, a set of values shared by the whole university community. Truth, Trust and Justice are at the core of NTU's shared values. As a student, it is important that you recognize your responsibilities in understanding and applying the principles of academic integrity in all the work you do at NTU. Not knowing what is involved in maintaining academic integrity does not excuse academic dishonesty. You need to actively equip yourself with strategies to avoid all forms of academic dishonesty, including plagiarism, academic fraud, collusion and cheating. If you are uncertain of the definitions of any of these terms, you should go to the academic integrity website for more information. On the use of technological tools (such as Generative AI tools), different courses / assignments have different intended learning outcomes. Students should refer to the specific assignment instructions on their use and requirements and/or consult your instructors on how you can use these tools to help your learning. Consult your instructor(s) if you need any clarification about the requirements of academic integrity in the course.

Policy (General)

You are expected to attend all lectures and tutorials, and attempt the midterm test and final exam. You are expected to actively participate in lecture and tutorial activities, which will contribute half of the marks for continuous assessment.

Policy (Absenteeism)

Absence Due to Medical or Other Reasons
If you are sick and unable to attend your class / Mid-terms, you have to:
1. Send an email to the instructor regarding the absence and request for a replacement class and make-up mid-terms.
2. Submit the original Medical Certificate* or official letter of excuse to administrator.
3. Attend the assigned replacement class (subject to availability) and make-up mid-terms.
* The medical certificate mentioned above should be issued in Singapore by a medical practitioner registered with the Singapore Medical Association

Policy (Others, if applicable)

Diversity and inclusion policy

Integrating a diverse set of experiences is important for a more comprehensive understanding of science.

It is our goal to create an inclusive and collaborative learning environment that supports a diversity of perspectives and learning experiences, and that honours your identities; including ethnicity, gender, socioeconomic status, sexual orientation, religion or ability.

To help accomplish this:
- If you are neuroatypical or neurodiverse, have dyslexia or ADHD (for example), or have a social anxiety disorder or social phobia;
- If you feel like your performance in the class is being impacted by your experiences outside of class;
- If something was said in class (by anyone, including the instructor) that made you feel uncomfortable;

Please speak to your teaching team, our school pastoral officer or a peer or senior (either in-person or via email) about how we can help facilitate your learning experience.

As a participant in course discussions, you should also strive to honour the diversity of your classmates. You can do this by: using preferred pronouns and names; being respectful of others opinions and actively making sure all voices are being heard; and refraining from the use of derogatory or demeaning speech or actions.

All members of the class are expected to adhere to the NTU anti-harassment policy. if you witness something that goes against this or have any other concerns, please speak to your instructors or a faculty member.