## Annexe A: New/Revised Course Content in OBTL+ Format

## **Course Overview**

| Course Over view   |                   |
|--|-------------------|
| Expected Implementation in Academic Year                   | AY2025-2026       |
| Semester/Trimester/Others (specify approx. Start/End date) | Semester 1        |
| Course Author  * Faculty proposing/revising the course     | Wu Guohua         |
| Course Author Email  | guohua@ntu.edu.sg |
| Course Title   | Probability       |
| Course Code  | MH2500            |
| Academic Units   | 4                 |
| Contact Hours  | 51                |
| Research Experience Components                             | Not Applicable    |

## Course Requisites (if applicable)

| Pre-requisites        | MH1100 AND MH1101 OR MH1800 AND MH1801 OR MH1101 AND MH110S OR<br>MH1100 AND MH111S OR MH1802 OR CY1601 OR MH1805 |
|-----------------------|---|
| Co-requisites         |   |
| Pre-requisite to      |   |
| Mutually exclusive to | MH1820, MH2814, SC2000  |
| Replacement course to |   |
| Remarks (if any)      |   |

#### **Course Aims**

This is a core mathematical course, aiming to develop your understanding of fundamental concepts in probability such as random variables, independence, basic probability distributions, and conditional expectations and conditional variances, the law of large numbers and the central limit theorem with applications. The course also prepares you for further courses in probability and statistics, such as MH2510, MH3500 and MH3512.

## Course's Intended Learning Outcomes (ILOs)

Upon the successful completion of this course, you (student) would be able to:

| ILO 1 | Calculate probabilities of events concerning discrete distributions by counting  |
|-------|--|
| ILO 2 | Calculate conditional probabilities with Bayes Theorem   |
| ILO 3 | Describe a probability distribution using cdf/pdf, including joint and marginal distributions  |
| ILO 4 | Identify the scenarios which certain probability distributions are appropriate to model  |
| ILO 5 | Calculate the expectation/variance/mg/quantiles of a probability distribution/random variable, including marginal distributions                                  |
| ILO 6 | Calculate the distribution of functions of random variables (especially the sum and quotient of two random variables) and the covariance of two random variables |
| ILO 7 | Prove or disprove that two random variables are independent  |
| ILO 8 | Calculate conditional expectations and conditional variances   |
| ILO 9 | Explain central limit theorem and its significance, and apply central limit theorem to approximate the probability of event concerning a sum of random variables |

#### **Course Content**

Events, probabilities, law of total probability, Bayes theorem

Independence events

Discrete distributions, continuous distributions

Joint distribution, marginal and conditional distribution for discrete variables

Joint distribution, marginal and conditional distribution for continuous variables

Functions of two or more random variables, order statistics, exponential distribution

Expectation, variance,

Covariance, correlation coefficient

Markov inequality, Chebyshev inequality

Conditional expectations, conditional variances and moment generating functions

Law of large numbers, central limit theorem with applications

### Reading and References (if applicable)

Textbook: [R1] Sheldon Ross. A First Course in Probability. Pearson, 2020. (10th edition.

ISBN: 978-0134753119)

NOTE: The above listing comprises the foundational readings for the course and more up-to-

date relevant readings will be provided when they are available.

## **Planned Schedule**

| Week<br>or<br>Session | Topics or Themes   | ILO           | Readings                     | Delivery<br>Mode | Activities |
|-----------------------|--|---------------|------------------------------|------------------|------------|
| 1                     | Events, probabilities, law of total probability, Bayes theorem | 1, 2,<br>6, 7 | [R1] §2.1-2.5, §3.1-3.2, 3.5 | In-person        |            |
| 2                     | Independence<br>events, discrete<br>distributions              | 3, 4          | [R1] §3.3-3.4                | In-person        |            |
| 3                     | discrete random<br>variables                                   | 3-5           | [R1] §4.1-4.2                | In-person        |            |
| 4                     | discrete random variables                                      | 3-5           | [R1] §4.1-4.2                | In-person        | Test 1     |
| 5                     | discrete random variables                                      | 3-5           | [R1] §4.7-4.10               | In-person        |            |
| 6                     | continuous<br>random<br>variables                              | 3, 6,<br>7    | [R1] §5.1-5.4                | In-person        |            |
| 7                     | continuous<br>random<br>variables                              | 3, 6,<br>7    | [R1] §5.5-5.6                | In-person        |            |
| 8                     | jointly<br>distributed<br>random<br>variables                  | 3, 6,         | [R1]§6.1-6.3                 | In-person        |            |
| 9                     | jointly<br>distributed<br>random<br>variables                  | 6             | [R1] §6.4-6.6                | In-person        |            |
| 10                    | expectations<br>involving<br>multiple random<br>variables      | 5-6           | [R1] §7.1-7.4                | In-person        | Test 2     |

| Week<br>or<br>Session | Topics or Themes                                  | ILO | Readings      | Delivery<br>Mode | Activities |
|-----------------------|---|-----|---------------|------------------|------------|
| 11                    | covariance and<br>correlation<br>coefficients     | 5-6 | [R1] §7.5-7.8 | In-person        |            |
| 12                    | Law of large<br>numbers, central<br>limit theorem | 8   | [R1] §8.1-8.4 | In-person        |            |
| 13                    | applications of<br>central limit<br>theorem       | 9   | [R2] §9.1-9.3 | In-person        |            |

# Learning and Teaching Approach

| Approach  | How does this approach support you in achieving the learning outcomes?   |
|-----------|--|
| Lectures  | Help you to understand the motivation and definitions of the concepts and notions, approaches to solving the problems in pursuant to learning outcomes |
| Tutorials | Develop communication and presentation skills, help you to understand better the concepts and notions, and techniques in problem solving               |

## **Assessment Structure**

Assessment Components (includes both continuous and summative assessment)

| No. | Component   | ILO   | Related PLO<br>or<br>Accreditation |    | Description<br>of<br>Assessment<br>Component | Team/Individual | Rubrics  | Level of<br>Understanding |
|-----|---|---|------------------------------------|----|--|-----------------|----------|---------------------------|
| 1   | Continuous<br>Assessment (CA):<br>Test/Quiz(Test 1)                                 | 1,<br>2,<br>3,<br>5                               |                                    | 20 |  | Individual      | Analytic | Relational                |
| 2   | Continuous<br>Assessment (CA):<br>Test/Quiz(Test 2)                                 | 5,<br>6,<br>7                                     |                                    | 20 |  | Individual      | Analytic | Relational                |
| З   | Summative<br>Assessment<br>(EXAM): Final<br>exam(Short Answer<br>Questions)         | 1,<br>2,<br>3,<br>4,<br>5,<br>6,<br>7,<br>8,<br>9 |                                    | 50 |  | Individual      | Analytic | Multistructural           |
| 4   | Continuous<br>Assessment (CA):<br>Class<br>Participation(Short<br>Answer Questions) | 1,<br>2,<br>3,<br>4,<br>5,<br>7,<br>8,<br>9       |                                    | 10 |  | Individual      | Holistic | Relational                |

#### Description of Assessment Components (if applicable)

The CA part consists of Test 1, Test 2 and Class Participation, and the two tests are allocated in Week 4 and Week 10 correspondingly. Topics covered in Test 1 are basics of probability, conditional probability and Bayes theorem. Test 2 covers marginal distribution, joint distribution, expectation, variance. The part of class participation will be conducted in the tutorials, where students are required to answer short questions. CA will contribute 50% to the assessment.

The Exam covers all topics of this course. The exam contributes 50% to the assessment.

#### Formative Feedback

Feedback will be given after each midterm on common mistakes and level of difficulty of the problems. For the final exam, comments on answers and common errors will also be given to students after the exams are marked. Feedback can also be given through discussion within tutorial lessons.

## NTU Graduate Attributes/Competency Mapping

This course intends to develop the following graduate attributes and competencies (maximum 5 most relevant)

| Attributes/Competency | Level    |  |
|-----------------------|----------|--|
| Adaptability          | Advanced |  |
| Curiosity             | Advanced |  |
| Critical Thinking     | Advanced |  |

### **Course Policy**

#### Policy (Academic Integrity)

Good academic work depends on honesty and ethical behaviour. The quality of your work as a student relies on adhering to the principles of academic integrity and to the NTU Honour Code, a set of values shared by the whole university community. Truth, Trust and Justice are at the core of NTU's shared values. As a student, it is important that you recognize your responsibilities in understanding and applying the principles of academic integrity in all the work you do at NTU. Not knowing what is involved in maintaining academic integrity does not excuse academic dishonesty. You need to actively equip yourself with strategies to avoid all forms of academic dishonesty, including plagiarism, academic fraud, collusion and cheating. If you are uncertain of the definitions of any of these terms, you should go to the academic integrity website for more information. On the use of technological tools (such as Generative Al tools), different courses / assignments have different intended learning outcomes. Students should refer to the specific assignment instructions on their use and requirements and/or consult your instructors on how you can use these tools to help your learning. Consult your instructor(s) if you need any clarification about the requirements of academic integrity in the course.

#### Policy (General)

You are expected to attend all classes punctually and take all scheduled tutorials and tests by due dates. You are expected to take responsibility to follow up with course notes, assignments and course related announcements. You are expected to participate in all discussions, presentations and activities. You are expected to attend the midterms. In case of medical leaves, you should provide a satisfying medical certificate on time.

#### Policy (Absenteeism)

Absence due to medical or other reasons. If you are sick and unable to attend a test you must:

- 1. Send an email to the instructor regarding the absence.
- 2. Submit the original Medical Certificate to an administrator.

The Medical Certificate mentioned above should be issued in Singapore by a medical practitioner registered with the Singapore Medical Association. There will be make-up opportunity for students who are absent from the tests with valid reason.

#### Policy (Others, if applicable)

Diversity and inclusion policy

Integrating a diverse set of experiences is important for a more comprehensive understanding of science.

It is our goal to create an inclusive and collaborative learning environment that supports a diversity of perspectives and learning experiences, and that honours your identities; including ethnicity, gender, socioeconomic status, sexual orientation, religion or ability.

#### To help accomplish this:

• If you are neuroatypical or neurodiverse, have dyslexia or ADHD (for example), or have a social anxiety disorder or social phobia;

- If you feel like your performance in the class is being impacted by your experiences outside of class;
- If something was said in class (by anyone, including the instructor) that made you feel uncomfortable;

Please speak to your teaching team, our school pastoral officer or a peer or senior (either in-person or via email) about how we can help facilitate your learning experience.

As a participant in course discussions, you should also strive to honour the diversity of your classmates. You can do this by: using preferred pronouns and names; being respectful of others opinions and actively making sure all voices are being heard; and refraining from the use of derogatory or demeaning speech or actions.

All members of the class are expected to adhere to the NTU anti-harassment policy. if you witness something that goes against this or have any other concerns, please speak to your instructors or a faculty member.

Appendix 1: Assessment Rubrics

Rubric for Tutorials: Participation and Short Answer Questions (10%)

| Criteria         | Standards           |                         |                        |  |
|------------------|---------------------|-------------------------|------------------------|--|
|                  | Fail standard (0-5) | Pass standard (6-8)     | High standard (9-10)   |  |
| Participation    | Lack participation  | Enough participation    | Active participations, |  |
| And Short Answer | or unsatisfactory   | and provide             | and provide correct    |  |
| Questions        | answers to short    | satisfactory answers to | answers to short       |  |
|                  | questions           | short questions         | questions              |  |

Rubric for Tests: Test 1 (20%, point-based marking)

| Criteria              | Standards   |   |   |
|-----------------------|---|---|---|
|                       | Fail standard   | Pass standard   | High standard   |
| Methods of approach   | Using methods that are irrelevant or do not apply to the given problem. Invoking theorems whose conditions are not satisfied. | Using relevant methods that help solve the problem. Invoking theorems whose conditions are satisfied. | Finding methods and utilizing theorems that are both relevant and effective |
| Validity of reasoning | Reasoning is logically invalid.   | Reasoning is logically valid.   | Reasoning is logically valid and effective.                                 |
| Clarity of argument   | Reasoning is poorly explained or not explained at all.  | Reasoning is clear but may contain some gaps.   | Reasoning is clear, precise with no or insignificant gaps.                  |

Rubric for Tests: Test 2 (20%, point-based marking)

| Criteria              | Standards   |   |   |
|-----------------------|---|---|---|
|                       | Fail standard   | Pass standard   | High standard   |
| Methods of approach   | Using methods that are irrelevant or do not apply to the given problem. Invoking theorems whose conditions are not satisfied. | Using relevant methods that help solve the problem. Invoking theorems whose conditions are satisfied. | Finding methods and utilizing theorems that are both relevant and effective |
| Validity of reasoning | Reasoning is logically invalid.   | Reasoning is logically valid.   | Reasoning is logically valid and effective.                                 |
| Clarity of argument   | Reasoning is poorly explained or not explained at all.  | Reasoning is clear but may contain some gaps.   | Reasoning is clear, precise with no or insignificant gaps.                  |

### Rubric for Examination: Final Exam (50%, point-based marking)

| Criteria              | Standards   |   |   |
|-----------------------|---|---|---|
|                       | Fail standard   | Pass standard   | High standard   |
| Methods of approach   | Using methods that are irrelevant or do not apply to the given problem. Invoking theorems whose conditions are not satisfied. | Using relevant methods that help solve the problem. Invoking theorems whose conditions are satisfied. | Finding methods and utilizing theorems that are both relevant and effective |
| Validity of reasoning | Reasoning is logically invalid.   | Reasoning is logically valid.   | Reasoning is logically valid and effective.                                 |
| Clarity of argument   | Reasoning is poorly explained or not explained at all.  | Reasoning is clear but may contain some gaps.   | Reasoning is clear, precise with no or insignificant gaps.                  |